NONSTEADY BURNING OF A SOLID PROPELLANT IN A GASEOUS OXIDIZER FLOW

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The burning of a solid propellant is investigated for nonsteady heat propagation in the induction zone. The equation of heat conduction in the propellant is solved in finite form for the case of a sharp change in burning rate; the time dependence of the temperature gradient at the propellant surface is obtained and used to investigate the mechanism of collapse of the diffusion flame above the surface. The combustion stability of a propellant burning in a channel with a large free volume is analyzed. The perturbations of the gas-dynamic quantities are related with the perturbations of the burning rate and hence with the properties of the induction zone in the solid phase. An analysis of the dispersion relation for the limiting case of propagation of acoustic waves in a stationary gas shows that the longitudinal acoustic perturbations that develop in the channel may grow with time, interacting with the heated subsurface layer of propellant.

1. The nonsteady burning of solid and liquid propellants is accompanied by a change in the state of the subsurface heating (induction) zone that is characterized by the temperature gradient at the surface. In the diffusion burning regime the burning rate is determined in the first approximation by the rate of supply of oxidizer to the diffusion flame and thus depends importantly on the parameters of the oxidizer gas flow bathing the surface of the propellant. The physicochemical properties of the propellant itself, its heat of gasification, specific heat, thermal conductivity, etc., have a much weaker influence on the burning rate, so that they can be neglected [1].

The nonsteady propagation of heat in the induction zone is described by the linear one-dimensional heat conduction equation

$$\frac{\partial\theta}{\partial\tau} - w(\tau)\frac{\partial\theta}{\partial\xi} = \frac{\partial^2\theta}{\partial\xi^2} \quad (1.1)$$

with the following initial and boundary conditions

$$\tau = 0,\ \theta = e^{-\xi},\ w = 1;\ \xi = 0,\ \theta = 1;\ \xi = \infty,\ \theta = 0 \quad (1.2)$$

Problem (1.1) and (1.2) is written in the dimensionless variables

$$\xi = \frac{u^\circ}{\varkappa}y, \quad \tau = \frac{u^{\circ 2}}{\varkappa}t, \quad \theta = \frac{T - T_0}{T_s - T_0}, \quad w(\tau) = \frac{u(t)}{u^\circ}$$

Here, t is time, y is the coordinate normal to the propellant surface (the propellant is located at $y > 0$), $\varkappa = \lambda_1/\rho_1 c_1$ is the thermal diffusivity of the propellant, u° is the steady-state burning rate maintained at $t < 0$, as a result of which the steady-state temperature distribution was established, T_S is the surface temperature of the propellant, T_0 is the temperature in the interior of the propellant. Steady-state values are denoted by a superscript °. The time dependence of the burning rate u(t) is given by the oxidizer flow conditions.

For a sharp change in burning rate $w(\tau) = k = \text{const}$ it is possible to find the solution of (1.1), (1.2) in analytic form using the Laplace transformation. We introduce the transformed temperature

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$$\theta(\xi, s) = \int_0^\infty \theta(\xi, \tau) e^{-s\tau} d\tau.$$

From Eq. (1.1) and conditions (1.2) we have

$$\theta(\xi, s) = \frac{\exp(-\xi)}{k + s - 1} + \frac{(k-1)\exp\left(-{}^1/_2 k + \sqrt{{}^1/_4 k^2 + s}\right)\xi}{s(k + s - 1)}.$$

Using the inversion formula, we obtain

$$\theta(\xi, \tau) = e^{-\xi-(k-1)\tau} + (k-1)e^{-{}^1/_2 k\xi} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{\exp\left(\tau s - \sqrt{{}^1/_4 k^2 + s}\,\xi\right)}{s(k+s-1)} ds \qquad (1.3)$$

In the complex plane s the line of integration in (1.3) passes to the right of the singularities of the integrand function, which are located at the points $s_1 = 0$ (pole), $s_2 = 1 - k$ (pole located on the negative or positive semiaxis depending on the value of k), $s_3 = -{}^1/_4 k^2$ (branch point). Using the standard method of obtaining the inverse transform [2], we find that the integral on the right side of (1.3) is equal to the sum of the residues of the integrand function at the poles s_1 and s_2 and to the integral along a branch cut in the complex plane s, which may conveniently be taken from the branch point s_3 along the real axis to $s \to -\infty$. As a result we obtain

$$2\theta(\xi, \tau) = 1 - \Phi(\eta_1) + e^{-k\xi}[1 + \Phi(\eta_2)] + e^{(1-k)\tau-\xi}[1 + \Phi(\eta_3)] - e^{(1-k)\tau+\xi}[1 + \Phi(\eta_4)] \qquad (1.4)$$

$$\eta_{1,2} = {}^1/_2 (k\tau^{1/2} \pm \xi\tau^{-1/2}), \quad \eta_{3,4} = {}^1/_2 ((k-2)\tau^{1/2} \pm \xi\tau^{-1/2})$$

$$\left(\Phi(\eta) = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-t^2} dt\right)$$

The dimensionless temperature gradient

$$\varphi = -\frac{\partial\theta}{\partial\xi}\bigg|_{\xi=0} = \frac{1}{2}\left\{k\left[1 + \Phi\left(\frac{k\tau^{1/2}}{2}\right)\right] - (k-2)e^{(1-k)\tau}\left[1 + \Phi\left(\frac{(k-2)\tau^{1/2}}{2}\right)\right]\right\} \qquad (1.5)$$

2. In [1] it is shown that for a diffusion flame to exist above the propellant the temperature gradient at the surface must be less than a certain critical value. At the critical gradient f_* the flow q of oxidizer to the diffusion flame, given by the hydrodynamic conditions, takes its limiting value [3]

$$\lambda_1 f_* = \alpha T_e + \nu q (c_1 T_s + Q) - \frac{\alpha E}{2R \ln(k_0 \sqrt{\rho D}/q)} \qquad (2.1)$$

$$\left(f = \frac{u^\circ}{\varkappa}(T_s - T_0)\varphi\right)$$

Here, the heat transfer coefficient α determines the heat flow from the diffusion flame to the gas and is calculated from the hydrodynamic conditions, T_e is the temperature of the external flow, ρ is the gas density, E and ν are the activation energy and stoichiometric coefficient of the chemical reaction in the flame, R is the gas constant, Q is the total energy release per unit mass of propellant, which is equal to the difference between the energy of the reaction in the flame and the heat of gasification of the propellant, k_0 is a dimensionless quantity that contains the total order of the reaction, D is the diffusion coefficient.

We will investigate the mechanism of collapse of the diffusion flame in the presence of a sharp increase in burning rate. In Fig. 1 we have plotted the qualitative dependence of the critical (curve 1) and steady-state $\lambda_1 f^\circ = \nu q c_1 (T_s - T_0)$ (straight line 2) gradients on the flow of oxidizer to the flame (the asymptote of curve 1 is the straight line $q = k_0\sqrt{\rho D}$). Let the steady-state burning of the propellant with oxidizer

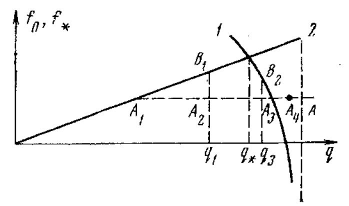

Fig. 1

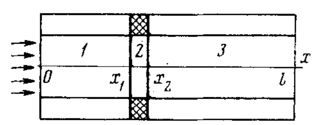

Fig. 2

flow q_0 be described by the point A_1. Following an instantaneous increase in burning rate the state of the propellant changes along the straight line A_1A parallel to the axis of abscissas, the temperature of the flame increasing. Then, at a constant burning rate corresponding to oxidizer flow q_1 the temperature gradient increases and a transition is made from the point A_2 to the new steady-state point B_1. The duration of this transient process τ_1 can be calculated by equating the gradient (1.5) to the steady-state gradient at the point B_1

$$k\,[1+\Phi\,(^1/_2 k\tau_1{}^{1/2})] - (k-2)\,e^{(1-k)\tau_1}\,[1+\Phi\,(^1/_2\,(k-2)\,\tau_1{}^{1/2})] = 2\nu q_2 / \rho_1 u^\circ \qquad (k = q_1/q_0) \tag{2.2}$$

If the increase in the flow of oxidizer to the flame is so great that the point describing the state of the propellant, moving along the straight line A_1A, intersects the curve 1 (for example, if it reaches the point A_4), then the diffusion flame will instantaneously collapse, since at the point of intersection the gradient becomes critical. If the increase in burning rate is such that the critical gradient is reached as the representative point moves along the vertical straight line (the transition $A_1 \rightarrow A_3 \rightarrow B_2$), then the diffusion flame does not collapse immediately, but only after the time required for the heated zone to relax from the point A_3 to the point B_2. This lag τ_2 can be calculated by equating the gradient (1.5) to the critical gradient at the point B_2

$$\begin{gathered} k\,[1+\Phi\,(^1/_2 k\tau_2{}^{1/2})] - (k-2)\,e^{(1-k)\tau_2}\,[1+\Phi\,(^1/_2\,(k-2)\,\tau_2{}^{1/2})] \\ = 2\left[\alpha T_e + \nu q_3\,(c_1 T_s + Q) - \frac{\alpha E}{2R\ln(k_0\sqrt{\rho D}/q_3)}\right] \Big/ \rho_1 u^\circ c_1\,(T_s - T_0) \\ (k = q_3/q_0) \end{gathered} \tag{2.3}$$

The maximum increase in burning rate k_*, at which the diffusion flame does not collapse, corresponds to the oxidizer flow q_* and can be calculated from the conditions of intersection of curves 1 and 2

$$\alpha T_e + \nu q_*\,(c_1 T_s + Q) - \frac{\alpha E}{2R\ln(k_0\sqrt{\rho D}/q_*)} = \nu q_* c_1\,(T_s - T_0) \qquad (k_* = q_*/q_0) \tag{2.4}$$

3. We will investigate the acoustic stability of steady-state combustion of a solid propellant in a cylindrical channel, through which flows a gaseous oxidizer (Fig. 2). We assume that in zone 2 ($x_1 \le x \le x_2$, $x_{1,2} = x_0 \pm {}^1\!/_2 b$), which is short as compared with the length of the channel, diffusion burning of the propellant takes place, while the rest of the channel (zones 1 and 3) does not contain propellant (the region occupied by the propellant is cross-hatched in the figure).

We employ the one-dimensional gas-dynamic equations of an ideal gas with heat and mass sources concentrated at the channel walls in zone 2; the strength of these sources depends on the parameters of the gas flow in accordance with the following expression for the mass burning rate (see [4])

$$m = B_0 c j^n \tag{3.1}$$

where c is the relative mass oxidizer concentration, B_0 and n are constants, and j is the mass velocity of the gas flow.

Our investigation is based on the model described in detail in [5]. On the steady-state solutions in zones 1 and 3 we superimpose small perturbations that depend on time as $\exp \beta t$, $\beta = \nu + i\omega$. Linearizing the equations in zones in 1 and 3 with respect to these perturbations, we obtain the solutions in the form of standing acoustic waves [5]

$$\begin{gathered} \delta v = \frac{1}{2a^\circ \rho^\circ}\left[A\exp\beta\left(t - \frac{x}{v^\circ + a^\circ}\right) + B\exp\beta\left(t - \frac{x}{v^\circ - a^\circ}\right)\right] \\ \delta p = \frac{1}{2}\left[A\exp\beta\left(t - \frac{x}{v^\circ + a^\circ}\right) - B\exp\beta\left(t - \frac{x}{v^\circ - a^\circ}\right)\right] \end{gathered} \tag{3.2}$$

Here, x is the coordinate along the channel axis, v is the gas velocity, p is pressure, a is the speed of sound, A and B are constants; the perturbation of the quantity ψ is denoted by $\delta\psi$, and here and in what follows steady-state values of the parameters are indicated by a superscript °. The perturbations of entropy S and oxidizer concentration are entrained by the gas flow

$$\delta S = C\exp\beta\left(t - \frac{x}{v^\circ}\right), \quad \delta c = H\exp\beta\left(t - \frac{x}{v^\circ}\right) \tag{3.3}$$

As the acoustical boundary conditions at the entrance section of the channel we take the conditions used in [5], namely,

$$x = 0, \ \ \rho v = \text{const}, \ \ \rho v c = \text{const} \tag{3.4}$$

Conditions (3.4) presuppose that the gas is supplied to the channel through a supersonic nozzle; accordingly, the rate of flow of the gas entering the channel is constant. Moreover, since the acoustic vibrations do not produce entropy perturbations of the same order of smallness, we assume that

$$x = 0, \ \ S = \text{const} \tag{3.5}$$

Assuming that the flow of gas through the supersonic nozzle, in which the channel ends, is quasi-stationary (i.e., that the period of the acoustic vibrations is large as compared with the time taken by a fluid element to move through the nozzle zone, and that the length of the entropy wave transported by the flow is much greater than the distance from the channel exit section to the throat of the nozzle at the channel outlet), we have (see [5])

$$x = l, \ \ M = \text{const} \ (\text{M is the Mach number}) \tag{3.6}$$

Linearizing (3.4)-(3.6), we obtain

$$\begin{gathered} x = 0, \ \ \frac{\delta v}{v^\circ} + \frac{1}{\gamma}\frac{\delta p}{p^\circ} = 0, \ \ \delta c = 0, \ \ \delta S = 0 \\ x = l, \ \ \delta M = 0 \end{gathered} \tag{3.7}$$

where γ is the ratio of specific heats. The boundary conditions at the channel inlet give

$$B_- = \frac{M_-^\circ + 1}{M_+^\circ - 1} A_-, \ \ C_- = 0, \ \ H_- = 0 \tag{3.8}$$

Here and in what follows quantities relating to zone 1 are denoted by a minus sign, quantities relating to zone 3 by a plus sign.

We note that in the absence of a combustion zone the flow is naturally stable. In fact, substituting (3.2) and (3.3) in (3.7) and using (3.8), we obtain

$$\begin{gathered} \nu = \frac{(M^{\circ 2} - 1)\, a^\circ}{2l} \ln \frac{(1 + M^\circ)\,[2 + (\gamma - 1)\, M^\circ]}{(1 - M^\circ)\,[2 - (\gamma - 1)\, M^\circ]} \\ \omega = \frac{\pi a^\circ}{l}(1 - M^{\circ 2})\, N, \ \ \ \ N = 0, 1, 2 \ldots \end{gathered}$$

Since $M^\circ < 1$ we have $\nu < 0$. The attenuation of the standing acoustic waves is caused by the dissipation of acoustic energy through the nozzle.

The gas flow in combustion zone 2 is described by the following equations used in [4] for the analysis of transient channel burning regimes

$$\begin{gathered} \frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} = \nu_1 \frac{2}{r} m \\ \frac{\partial \rho c}{\partial t} + \frac{\partial \rho c v}{\partial x} = -\nu_2 \frac{2}{r} m, \ \ \ \ \frac{\partial \rho v}{\partial t} + \frac{\partial}{\partial x}(p + \rho v^2) = 0 \\ \frac{\partial}{\partial t} \rho \left(\frac{1}{\gamma - 1}\frac{p}{\rho} + \frac{v^2}{2}\right) + \frac{\partial}{\partial x} \rho v \left(\frac{\gamma}{\gamma - 1}\frac{p}{\rho} + \frac{v^2}{2}\right) = \frac{2}{r}(Qm - \lambda_1 f) \end{gathered} \tag{3.9}$$

where ν_1 and ν_2 are stoichiometric coefficients, and r is the channel radius.

Following [5] we replace zone 2 with a surface of strong discontinuity located at $x = x_0$. On the left of this surface the flow perturbations coincide with the perturbations at $x = x_0 - b/2$, while on the right of the surface $x = x_0$ they coincide with the corresponding quantities at $x = x_0 + b/2$. When we replace the real combustion zone with a surface of strong discontinuity, we must assign to that surface all the important properties of zone 2; consequently, the solutions (3.2) and (3.3) on the left and right of that zone are coupled by conditions that take into account the transfer of mass and thermal energy associated with combustion. We will obtain one of these conditions. Integrating the first of equations (3.9) with respect to x across the combustion zone, we have

$$\frac{\partial}{\partial t} \int_{x_1}^{x_2} \rho\, dx + \rho_+ v_+ - \rho_- v_- = \nu_1 \frac{2}{r} B_0 \int_{x_1}^{x_2} c j^n\, dx \tag{3.10}$$

The integral on the left of (3.10) is equal to zero, since mass transfer takes place in a region fixed relative to the channel walls and of short extent in the direction of the channel axis ($b \to 0$). We make the simplifying assumption that the mass burning rate depends on the flow parameters at $x = x_0 - 0$. Then (3.10) takes the form

$$x = x_0, \quad \rho_+ v_+ - \rho_- v_- = \nu_1 \frac{2}{r} B_0 c_- j_-^{n} b$$

In the same way we obtain the conditions at the surface of discontinuity for the other three equations (3.9). Linearizing the relations obtained with respect to the small corrections and using (3.8) and the relations [6] between a small change in the burning rate (expressed by means of (3.1) in terms of the perturbations of the oxidizer concentration and the mass velocity of the gas flow) and the perturbation of the temperature gradient at the propellant surface written in the form

$$\frac{\delta f}{f^\circ} = \frac{\sqrt{1 + 4\beta t_0} - 1}{2\beta t_0} \frac{\delta m}{m^\circ}$$

where $t_0 = \varkappa / u^{\circ 2}$ is the characteristic response time of the thermal zone, we obtain the following condition relating the solutions in zones 1 and 3

$$\rho_+^\circ \delta v_+ + \frac{v_+^\circ}{a_+^{\circ 2}} \delta p_+ - \frac{j_+^\circ}{c_p} \delta S_+ = \Delta_1 \left(\rho_-^\circ \delta v_- + \frac{v_-^\circ}{a_-^{\circ 2}} \delta p_- \right)$$

$$\rho_+^\circ c_+^\circ \delta v_+ + j_+^\circ \delta c_+ + \frac{v_+^\circ c_+^\circ}{a_+^{\circ 2}} \delta p_+ - \frac{j_+^\circ c_+^\circ}{c_p} \delta S_+ = \Delta_2 \left(\rho_-^\circ \delta v_- + \frac{v_-^\circ}{a_-^{\circ 2}} \delta p_- \right)$$

$$(1 + M_+^{\circ 2}) \delta p_+ + 2 j_+^\circ \delta v_+ - \frac{j_+^\circ v_+^\circ}{c_p} \delta S_+ = (1 - M_-^{\circ 2}) \delta p_- + 2 j_-^\circ \delta v_- \tag{3.11}$$

$$v_+^\circ \left(\frac{\gamma}{\gamma - 1} + \frac{1}{2} M_+^{\circ 2} \right) \delta p_+ + \rho_+^\circ a_+^{\circ 2} \left(\frac{1}{\gamma - 1} + \frac{3}{2} M_+^{\circ 2} \right) \delta v_+ - \frac{v_+^{\circ 2} j_+^\circ}{2 c_p} \delta S_+$$

$$= v_-^\circ \left(\frac{\gamma}{\gamma - 1} + \frac{1}{2} M_-^{\circ 2} + \frac{\Delta_3}{\gamma - 1} \right) \delta p_- + \rho_-^\circ a_-^{\circ 2} \left(\frac{1}{\gamma - 1} + \frac{3}{2} M_-^{\circ 2} + \frac{\Delta_3}{\gamma - 1} \right) \delta v_-$$

$$\Delta_1 = 1 + 2\nu_1 n \frac{b}{r} \frac{m_-^\circ}{j_-^\circ}, \quad \Delta_2 = c_-^\circ - 2\nu_2 n \frac{b}{r} \frac{m_-^\circ}{j_-^\circ}, \quad \Delta_3 = z_1 \left(1 - z_2 \frac{\sqrt{1 + 4\beta t_0} - 1}{2\beta t_0} \right)$$

$$z_1 = 2n \frac{b m_-^\circ Q}{r j_-^\circ c_p T_-^\circ}, \quad z_2 = \frac{c_1 (T_s - T_0)}{Q}$$

These relations hold at $x = x_0$. Here, c_p is the specific heat of the gas at constant pressure, z_1 is a parameter characterizing the strength of the sources in the combustion zone, and z_2 is the ratio of the heat stored in the solid propellant to the total reaction energy. To relations (3.11) we add the boundary condition (3.7) at the channel exit written in the form

$$x = l, \quad \frac{\gamma - 1}{\gamma} \frac{\delta p_+}{p_+^\circ} + \frac{\delta S_+}{c_p} - 2 \frac{\delta v}{v_+^\circ} = 0 \tag{3.12}$$

We now substitute the solutions (3.2) and (3.3) obtained above in (3.11) and (3.12), using relations (3.8). For purposes of a stability analysis the second of equations (3.11), which gives H_+ in terms of A_-, can be omitted, since the quantity H_+ occurs only in that equation. (This is a consequence of the fact that the perturbations of the oxidizer concentration, generated in the combustion zone and entrained by the gas flow, do not interact with the perturbations of pressure and velocity at the channel exit and in region 3.) As a result, we obtain a system of three equations expressing the quantities A_+, B_+, and C_+ in terms of the constant A_-, which remains indeterminate. The condition of solvability of this system of equations, which consists in its determinant being equal to zero, is very clumsy and not entirely amenable to investigation. Accordingly, we will consider the case of small Mach numbers, namely, $M_-^\circ, M_+^\circ \ll 1$. Retaining only terms of zero order in M_-° and M_+° and assuming that the quantities a_-°, a_+° are finite (acoustic vibrations in the stationary gas) and that $\nu > 0$ (unstable roots), we obtain the following equation for β:

$$\operatorname{th} \beta t_+ \operatorname{cth} \beta t_- = -d \left[1 + z_1 \left(1 - z_2 \frac{\sqrt{1 + 4\beta t_0} - 1}{2\beta t_0} \right) \right] \tag{3.13}$$

$$(d = a_-^\circ / a_+^\circ)$$

Here, $t_- = x_0 / a_-^\circ$, $t_+ = (l - x_0)/a_+^\circ$ are the propagation times of the acoustic wave in zones 1 and 3, respectively.

If the propagation times of the acoustic wave in zones 1 and 3 are equal, the solution of (3.13) is easily found

$$\nu t_0 = z_1 z_2 \frac{z_1 z_2 - 1 - d}{(1 + z_1 + d)^2}, \quad \omega = 0 \tag{3.14}$$

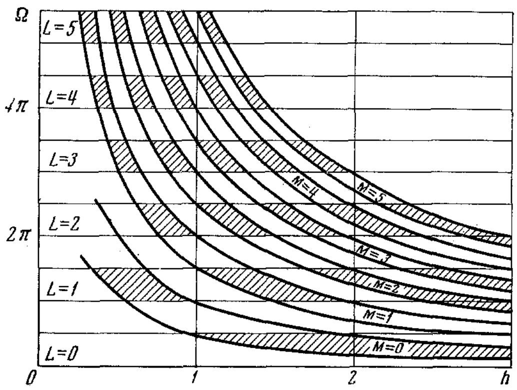

Fig. 3

For real values of z_1 and z_2 the numerator of the fraction in (3.15) is negative and $\nu < 0$; accordingly, at $t_- = t_+$ there are no solutions with $\nu > 0$.

If the nonstationary character of the subsurface heating zone, i.e., the reorganization associated with fluctuations of the burning rate, is disregarded, then the perturbations are damped and combustion is stable. In fact, in this case Eq. (3.13) takes the form

$$\operatorname{th} \beta t_+ \operatorname{cth} \beta t_- = -d(1+z_1)$$

Separating the real and imaginary parts, we obtain

$$\frac{\operatorname{sh} 2\nu t_- \operatorname{sh} 2\nu t_+ + \sin 2\omega t_- \sin 2\omega t_+}{(\operatorname{ch} 2\nu t_- - \cos 2\omega t_-)(\operatorname{ch} 2\nu t_+ + \cos 2\omega t_+)} = -d(1+z_1) \tag{3.15}$$

$$\sin 2\omega t_+ = \operatorname{sh} 2\nu t_+ \sin 2\omega t_- / \operatorname{sh} 2\nu t_- \tag{3.16}$$

Substituting the expression for $\sin 2\omega t_+$ from (3.16) in (3.15), we see that the left side of (3.15) is positive at $\nu > 0$. Since the right of this equation is negative, system of equations (3.15), (3.16) does not have solutions in the region $\nu > 0$.

We will now show that taking the nonstationary process into account leads to the appearance of unstable roots. For this purpose we consider Eq. (3.13), assuming that

$$\nu t_- \ll 1, \quad \nu t_+ \ll 1, \quad \omega \gg \nu, \quad \omega t_0 \gg 1 \tag{3.17}$$

Conditions (3.17) make it possible to substitute for (3.13) the following equation:

$$\frac{\sin 2\omega t_+ \sin 2\omega t_- + i\nu (t_- \sin 2\omega t_+ - t_+ \sin 2\omega t_-)}{4\cos^2 \omega t_+ \sin^2 \omega t_-} = -d\left[1 + z_1\left(1 - z_2 \frac{1-i}{\sqrt{2\omega t_0}}\right)\right] \tag{3.18}$$

In obtaining (3.18) we also made use of the conditions

$$\sin 2\omega t_- \neq 0, \quad \sin 2\omega t_+ \neq 0, \quad \nu t_- \ll |\sin \omega t_-| \tag{3.19}$$

$$\nu t_+ \ll |\sin \omega t_+|, \quad \nu^2 t_- t_+ \ll |\sin 2\omega t_- \sin 2\omega t_+|$$

Conditions (3.17) and (3.19) are subject to verification after the roots have been found.

Separating the real and imaginary parts in (3.18) and neglecting the quantity $z_1 z_2/\sqrt{2\omega t_0}$ as compared with unity by virtue of one of conditions (3.17), we obtain

$$F(\omega) = \operatorname{ctg} \omega t_- \operatorname{tg} \omega t_+ = -d_1^2 \quad (d_1^2 = d(1+z)) \tag{3.20}$$

$$\nu = d_2^2 \cdot \frac{\sin^2 \omega t_- \cos^2 \omega t_+}{t_+ \sin 2\omega t_- - t_- \sin 2\omega t_+} \qquad d_2^2 = \sqrt{\frac{2}{\omega t_0}}\, d z_1 z_2 \tag{3.21}$$

It is easy to show that the quantity ν is positive if the root of Eq. (3.20) satisfies the conditions $\tan \omega t_- > 0$, $\tan \omega t_+ < 0$, which we rewrite as follows:

$$\pi L < \Omega < \tfrac{1}{2}\pi(1+2L), \quad \frac{\pi(1+2M)}{2h} < \Omega < \frac{\pi(1+M)}{h} \tag{3.22}$$

$$(\Omega = \omega t_-; \quad h = t_+/t_-; \quad L, M = 0, 1, 2 \ldots)$$

Figure 3 represent the plane $h\Omega$, on which we have plotted the regions defined by inequalities (3.22) for various L and M; they constitute four-sided figures bounded by two straight lines $\Omega = \pi L$, $\Omega = \frac{1}{2}\pi(1+2L)$ and two hyperbolas $\Omega = \pi(1+2M)/2h$, $\Omega = \pi(1+M)/h$. In order to determine the values of Ω which, for a specific h, give $\nu > 0$, it is necessary to draw a straight line parallel to the ordinate axis, as has been done in Fig. 3 for $h = 2$; the region of required Ω lies between the ordinates of the upper and lower points of intersection of this straight line and the sides of a given quadrangle. For $h = 1$ there are no such regions, since the straight line $h = 1$ does not intersect any of the shaded figures; for any other h there are infinitely many (for $h < 1$ the projections on the axis of abscissas of the regions (3.22), constructed for fixed M, span all values of h owing to the overlapping of the projections of adjacent figures; all values of $h > 1$ are spanned by the projections on the axis of abscissas of the quadrangles constructed for fixed L).

We will now show that in any such region, found for a given h, there is a root of Eq. (3.20). In fact, as $\Omega \to \pi L + 0$ and $\Omega \to \pi(1+2M)/2h + 0$ the function $F(\Omega) \to -\infty$, and as $\Omega \to \frac{1}{2}\pi(1+2L) - 0$ and $\Omega \to \pi(1+M)/h - 0$ $F(\Omega) \to -0$. Hence as Ω varies within a given region, the function necessarily takes a value equal to $-d_1^2$.

Thus, for any $h \neq 1$ there exist infinitely many roots of equation (3.14) with $\nu > 0$. By selecting among them roots with a sufficiently large value of ω it is possible to satisfy conditions (3.17) and (3.19).

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